CHAPTER 2 REVIEW ACTIVITY

Precision, Accuracy, and Significant Figures

No measurement of a physical quantity is absolutely certain. In other words, all measurements include a degree of uncertainty. The two main causes of uncertainty are (a) the skill and care of the person making the measurement and (b) the limitations of the measuring instrument.

The two values in the illustration give measurements with different degrees of uncertainty. For both rulers, the measurement indicated by the arrow is clearly between 2 cm and 3 cm. But Ruler B gives the additional information that the length is between 2.7 cm and 2.8 cm. Most persons who use Ruler A would probably estimate a reading of from 2.7 cm to 2.9 cm. The average reading might then be 2.8 cm, with a maximum uncertainty of ±0.1 cm, or 2.8 ±0.1 cm. Those who use Ruler B would probably estimate a reading of 2.77 cm, with a maximum uncertainty of ±0.01 cm, or 2.77 ±0.01 cm. Thus we say that with Ruler A the measurement is known to a precision of ±0.1 cm. Similarly, the precision attainable with Ruler B is ±0.01 cm. In general, precision refers to the reproducibility of a measurement.

Suppose the scale on Ruler B had been designed to measure millimeters rather than centimeters, but the scale had been labeled “cm” by mistake. Although length measurements with the ruler would be precise to ±0.01 unit, the actual measurements would be off by a factor of 10. Thus, the accuracy of the ruler would be very poor. Accuracy indicates how close a measurement is to the true or accepted value.

The measurements made with Rulers A and B include one estimated, or uncertain, figure. Measurements that include one uncertain figure in addition to those known with certainty are made up of significant figures. Thus, 2.8 cm consists of two significant figures, and 2.77 cm consists of three.

Measurements are often used to calculate other quantities. For example, the length and width of a surface might be used to find its area \((L \times W = A)\). It is necessary to be able to estimate the uncertainty in a calculated result. Suppose all measurements are given in significant figures. Then the number of significant figures in a result calculated from those measurements can be found by using the following simple rules.

**Multiplication and Division.** The number of significant figures in a product or quotient obtained from measured quantities is the same as the number of significant figures in the quantity having the smaller number of significant figures.

**Addition and Subtraction.** Round the sum or difference so that it has the same number of decimal places as the quantity having the least number of decimal places.

**Example**

1. Find the area of a rectangle whose sides measure 1.2 cm and 4.56 cm.
2. Find the perimeter of this rectangle.
Precision, Accuracy, and Significant Figures (continued)

Solution

1. Area = length \times width
   = 1.2 \text{ cm} \times 4.56 \text{ cm}
   = 5.472 \text{ cm}^2, \text{ rounded to } 5.5 \text{ cm}^2

The answer can only be given with two significant figures, according to the rules for multiplication.

2. Perimeter = \text{sum of sides}
   = 1.2 \text{ cm} + 1.2 \text{ cm} + 4.56 \text{ cm} + 4.56 \text{ cm}
   = 11.52 \text{ cm}, \text{ rounded to } 11.5 \text{ cm}

The answer can only be given with one decimal place, according to the rules for addition.

Answer the following questions.

1. Look at the figure. Which of the following numbers should be used to indicate the length of the object? 2 cm, 2.2 cm, 2.20 cm, 2.200 cm, 2.5 cm, 3 cm

   ![Metric ruler]

   1. 

2. In which way might the uncertainty of the above answer be indicated using \pm notation?

   \begin{align*}
   2 \pm 1 \text{ cm} & \quad 2.2 \pm 1 \text{ cm} \quad 2.2 \pm 0.1 \text{ cm} \\
   2.2 \pm 0.01 \text{ cm} & \quad 2.20 \pm 0.1 \text{ cm} \quad 2.20 \pm 0.01 \text{ cm} \\
   2.20 \pm 0.001 \text{ cm} & \quad 2.200 \pm 0.01 \text{ cm} \quad 2.200 \pm 0.001 \text{ cm} \\
   2.5 \pm 0.1 \text{ cm} & \quad 2.5 \pm 0.01 \text{ cm} \quad 3 \pm 1 \text{ cm} \\
   3 \pm 0.1 \text{ cm} & \\
   \end{align*}

3. How many significant figures are in the measurement 40.04 cm²?

4. How many significant figures are in the measurement 400.000 cm²?

5. How many significant figures are in the measurement 3.15 cm²?

6. If the measurement in Question 5 were multiplied by another number, what is the maximum number of significant figures the product could have?

7. If the measurement in Question 5 were added to another number, what is the maximum number of significant decimal places the sum could have?
Section 2.4 Significant Figures in Measurements

Re-read Section 2.4 in your text

Regardless of the quality of any measuring device, there is always a degree of uncertainty in its measurements. Numbers that represent measured values consist of digits whose values are known with certainty and a final digit which is always an approximation or best guess of the true value. For example, suppose a student uses a meter stick divided into centimeters to measure a piece of wood and finds that the wood is between 9 and 10 cm long. The student decides that the piece of wood is 9.5 cm long. The measurement of 9.5 cm consists of two significant figures. The first digit, 9, is known with certainty but the last digit is just an estimate, and is therefore uncertain.

If the student measures the same piece of wood again, this time using a meter stick that is divided into millimeters, the second measurement would be more accurate than the first. But there would still be uncertainty in the results. Suppose that the student finds the length of the stick to be between 9.50 and 9.60 cm and decides the length is 9.56 cm. In the value of 9.56 cm, both the 9 and the 5 are known with certainty, but the 6 is just an estimated number and is uncertain.

In summary, significant figures include numbers whose digits are known with certainty and a final digit that is only an estimate. Zeros often are only placeholders in figures and are not always significant. To understand how to calculate the number of significant figures from measurements, review the following rules:

1. All nonzero digits are significant.
   23.5 g, 123 g, and 0.467 g all have three significant figures.

2. Zeros between nonzero digits are significant.
   1036 mL, 10.07 mL, and 0.1066 mL all have four significant figures.

3. Zeros in front of all nonzero digits are not significant; they act as placeholders. If a zero can be dropped when a measurement is written in scientific notation, it is only a placeholder and therefore is not significant.
   0.0004302 can be written as $4.302 \times 10^{-4}$. There are only four significant figures in this number.
   0.0412 can be written as $4.12 \times 10^{-2}$. There are only three significant figures in this number.

4. At the end of a number, zeros to the right of a decimal point are always significant in measured values.
   1203.0 mm, 617.00 mm, and 8090.0 mm all have five significant figures. To avoid ambiguity, the measurements should be written in scientific notation: $1.2030 \times 10^3$ mm, $6.1700 \times 10^2$ mm, and $8.0900 \times 10^3$ mm.

Apply

1. Write each measurement in scientific notation and determine the number of significant figures in each. Underline the digit that is the least certain.
   a. 12 305 L
   b. 0.0034 g
   c. 1200.1 cm
   d. 1000 mL
   e. 0.078804 mg
Section 2.5 Significant Figures in Calculations

Re-read Section 2.5 in your text

A calculated answer can never be more certain than the most uncertain piece of information used in making the calculation. For instance, suppose you are asked to find the product of two numbers, 4.13 and 52.39. Assume that the last digits in both numbers are estimated values. The product of 4.13 and 52.39 is 216.3707. But this calculated value cannot be more precise than the original measured data. The first number has three significant digits. The second number has four significant digits. The product must be written with the same number of significant figures as the shorter of the original numbers and therefore must be rounded off to 216.

The number of significant figures in an answer from a multiplication or division calculation are determined differently than the number of significant figures in an answer from an addition or subtraction calculation. In multiplication and division calculations, the answer is rounded off to the number of significant figures in the measurement with the least number of significant figures. For example:

$$3.1 \times 0.006 = 0.0186$$

in significant figures this answer is 0.02

The number 0.006 contains only one significant figure. Remember, if a zero is dropped when the figure is put into scientific notation, the zero is not significant. Since 0.006 can be rewritten as $6 \times 10^{-3}$, only the 6 is significant. The answer 0.0186 must be rounded off to 0.02 to reflect the answer with only one significant figure.

For addition and subtraction calculations, the answer should be rounded off to the same number of digits in the decimal places as the measurement with the least number of digits in the decimal places.

For example:

| 123.45 | This has the least number of digits in the decimal place. |
| 708.003 | The last two measurements are more precise but the answer cannot be more precise |
| $+1.005$ | than the least precise measurement. |
| 832.458 | The last digit needs to be rounded off to reflect the correct number of digits in the decimal place. |
| 832.46 | is the correct expression for the answer. |
| 782.004 | |
| $-10.91$ | This has the least number of digits in the decimal place. |
| 771.094 | Round off 771.094 to two digits in the decimal place. |
| 771.09 | is the correct expression for the answer. |

Apply

1. Do the following operations and give your answer with the correct number of significant figures.

   a. $2.3 \text{ cm} \times 1.35 \text{ cm} \times 1.55 \text{ cm} =$
   b. $0.00820 \text{ g} / 0.21 \text{ mL} =$
   c. $0.0035 \text{ cm} + 1.23 \text{ cm} + 1.005 \text{ cm} =$
   d. $23.678 \text{ mm} - 16.3 \text{ mm} =$
   e. $1000.0 \text{ g} / 20 \text{ mL} =$
   f. $8.79 \times 10^{-3} \text{ m} - 3.11 \times 10^{-4} \text{ m} =$
   g. $7.11 \times 0.003 =$
   h. $(2.15 \times 10^2)(3.2 \times 10^7) =$
   i. $0.057 / 0.300 =$
   j. $1.8 \times 10^{-3} + 2.2 \times 10^{-4} + 3.46 \times 10^{-2} =$
Section 2.2  Accuracy and Precision

*Re-read Section 2.2 in your text*

Sheila performed an experiment in order to understand the difference between accuracy and precision. She weighed herself using three different scales. She used a bathroom scale, the scale in the nurse’s office at school, and a scale in the gym. She recorded the following measurements:

<table>
<thead>
<tr>
<th>Bathroom Scale</th>
<th>Nurse’s Scale</th>
<th>Gym Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>144</td>
<td>155</td>
<td>139</td>
</tr>
<tr>
<td>134</td>
<td>153</td>
<td>138</td>
</tr>
<tr>
<td>139</td>
<td>154</td>
<td>140</td>
</tr>
</tbody>
</table>

She observed that the measurements from the bathroom scale were not very close in value to each other and therefore not very precise. Unlike the bathroom scale, the scale from the nurse’s office gave results that were reproducible; but since Sheila knew she did not weigh that much, these measurements were not accurate. The scale from the gym gave results that were both reproducible and close to her true weight, 139 pounds; these measurements were both accurate and precise.

Accuracy is how close a single measurement is to the true value of whatever is being measured. Precision is how close several measurements are to each other. Precise measurements have repeatability.

**Apply**

The following measurements were made on a piece of copper whose true mass is known to be 1.55 grams: 1.53 g, 1.40 g, 1.65 g, and 1.67 g.

1. Which two measurements are the most precise? Why are they the most precise?

2. Which measurement is the most accurate? Why is it the most accurate?

3. What are some of the steps you could take during an experiment to make sure that your measurements are both accurate and precise?

Section 2.3  Scientific Notation

*Re-read Section 2.3 in your text*

Scientific notation is used in many fields of science in order to make very large or small numbers easier to use. In scientific notation, a number is written as the product of two numbers: a coefficient and a power of ten to indicate the location of the decimal point.